Highly Fault-Tolerant Routings and Fault-Induced Diameter of Some Cartesian Digraphs

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The *Cartesian product digraph*, denoted by $G_1 \square G_2 \square \ldots \square G_n$, of $n$ digraphs $G_1, G_2, \ldots, G_n$ is the digraph with the vertex-set $V(G_1) \times V(G_2) \times \ldots \times V(G_n)$, and an arc from a vertex $x = x_1 x_2 \ldots x_n$ to another vertex $y = y_1 y_2 \ldots y_n$, where $x_i, y_i \in V(G_i)$ for each $i = 1, 2, \ldots, n$, iff they differ in exactly one coordinate, and for this coordinate, say $j^{th}$, there is an arc from the vertex $x_j$ to the vertex $y_j$ in $G_j$. 
Example: $\vec{C}_3 \square \vec{C}_4$
Graph can be viewed as digraph, symmetric digraph

A graph can be thought of as a particular digraph, *symmetric digraph*, in which there is a pair of symmetric arcs corresponding to each edge.

graph $K_2$  
symmetric digraph $K_2$
Definitions
Previous Results
Our Results
Further Problem

Cartesian Product Digraph
Digraph and Graph
Surviving Route Graph (幸存路由图)
Property $\mathcal{P}_k$

For a routing $\rho$ and a fault-set $F$ such that $G - F$ is strongly connected, the \textit{surviving route graph}, denoted by $R(G, \rho)/F$, is a digraph with the same vertex set as $G - F$, and a vertex $x$ being adjacent to another vertex $y$ iff $\rho(x, y)$ avoids $F$. 

Highly Fault-Tolerant Routings and Fault-Induced Diameter
\[ \rho(i, j) = \text{the shortest directed}(i, j)-\text{path} \]
We say a digraph $G$ has the property $\mathcal{P}_k$ if $G$ satisfies the following two conditions.

1. $\forall (x, y) \in V(G) \times V(G) (x \neq y)$, there are $k$ internally-disjoint directed $(x, y)$-paths such that one of them is a shortest directed $(x, y)$-path and each of others can be a concatenation of at most three shortest directed paths.

2. $\forall x \in V(G)$, there are $k$ directed cycles that contain $x$ and are vertex-disjoint except for $x$ such that each of them can be a concatenation of at most three shortest directed paths among which there is one that starts from $x$. 

Highly Fault-Tolerant Routings and Fault-Induced Diameter
Every arc represents a shortest directed path in $G$.

(1) of $\mathcal{P}_k$

(2) of $\mathcal{P}_k$
(D. Dolev et al., 1984) \( D(R(\{K_2 \Box K_2 \Box \cdots \Box K_2\}, \rho)/F) \leq 3 \) for any minimal routing \( \rho \) and \(| F | < n \).

(K. Wada et al., 1997) \( D(R(\{K_d \Box K_d \Box \cdots \Box K_d\}, \rho)/F) \leq 3 \) for any minimal routing \( \rho \) and \(| F | < n(d - 1) \), where \( d \geq 2 \).

(J.-M. Xu, 1998) \( D(R(\{\vec{C}_{d_1} \Box \vec{C}_{d_2} \Box \cdots \Box \vec{C}_{d_n}\}, \rho)/F) \leq 3 \) for any minimal routing \( \rho \) and \(| F | < n \), where \( \vec{C}_{d_i} \) is a directed cycle of order \( d_i \geq 2 \) for each \( i = 1, 2, \ldots, n \).
Theorem 1. \[ D(R(G_1 \Box G_2 \Box \ldots \Box G_n, \rho)/F) \leq 3 \]
for any minimal routing \( \rho \) and \( \|F\| = \max\{|F'| : F' \subseteq F \text{ and } F' \text{ contains no pair of symmetric arcs}\} \).

Each pair of symmetric arcs in \( F \) is calculated only once in \( \|F\| \).
Previous results viewed as corollaries

- **Corollary 2** (D. Dolev et al., 1984)
  \[ D(R(K_2 \square K_2 \square \ldots \square K_2), \rho)/F) \leq 3 \text{ for any minimal routing } \rho \text{ and } |F| < n. \]

- **Corollary 3** (K. Wada et al., 1997)
  \[ D(R(K_d \square K_d \square \ldots \square K_d), \rho)/F) \leq 3 \text{ for any minimal routing } \rho \text{ and } |F| < n(d - 1), \text{ where } d \geq 2. \]

- **Corollary 4** (J.-M. Xu, 1998)
  \[ D(R(C_{d_1} \square C_{d_2} \square \ldots \square C_{d_n}, \rho)/F) \leq 3 \text{ for any minimal routing } \rho \text{ and } |F| < n, \text{ where } d_i \geq 2 \text{ for each } i = 1, 2, \ldots, n. \]
**Corollary 5.** \[ D(R(K_{d_1} \square K_{d_2} \square \ldots \square K_{d_n}, \rho)/F) \leq 3 \] for any minimal routing \( \rho \) and \( |F| < \sum_{i=1}^{n} d_i - n \), where \( d_i \geq 2 \) for each \( i = 1, 2, \ldots, n \).

**Corollary 6.** \[ D(R(FP_n, \rho)/F) \leq 3 \] for any minimal routing \( \rho \) and \( |F| < 3n \), where \( FP_n \) is the Folded Petersen network of dimension \( n \), i.e. \( FP_n = P \square P \square \times \square P \).

**Corollary 7.** \[ D(R(HP_n, \rho)/F) \leq 3 \] for any minimal routing \( \rho \) and \( |F| < n \), where \( HP_n \) is the Hyper Petersen network of dimension \( n \), i.e. \( HP_n = K_2 \square K_2 \square \times \square K_2 \square P \).

**Corollary 8.** \[ D(R(G_1 \square G_2 \square \ldots \square G_n, \rho)/F) \leq 3 \] for any minimal routing \( \rho \) and \( |F| < n \) if \( G_i \) is a connected graph of order at least two and has a unique minimal routing for each \( i = 1, 2, \ldots, n \).
Koichi Wada, Optimal Fault-Tolerant Routings on Surviving Route Graph Model
http://citeseerx.ist.psu.edu/viewdoc-summary?doi=10.1.1.29.1153

Two directions in surviving route graph model

1. The one is that for arbitrary graphs we develop routings that minimize the diameter of the surviving route graph.

2. The other is that we develop both graphs and routings on these graphs that make the diameter of the surviving route graph small.
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谢谢！
There Is No Largest Prime Number

The proof uses *reductio ad absurdum*.

**Theorem**

*There is no largest prime number.*

**Proof.**

1. Suppose \( p \) were the largest prime number.
2. Let \( q \) be the product of the first \( p \) numbers.
3. Then \( q + 1 \) is not divisible by any of them.
4. Thus \( q + 1 \) is also prime and greater than \( p \).
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