Roman domination on 2-connected graphs

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June, 2009
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Definition

1. A Roman dominating function of a graph $G$ is a function $f : V(G) \rightarrow \{0, 1, 2\}$ such that whenever $f(v) = 0$, there exists a vertex $u$ adjacent to $v$ such that $f(u) = 2$.

2. The weight of $f$ is $w(f) = \sum_{v \in V(G)} f(v)$.

3. The Roman domination number $\gamma_R(G)$ of $G$ is the minimum weight among all Roman dominating functions of $G$. 
Roman domination on 2-connected graphs

Introduction

Origin

In ancient Roma, Emperor Constantine would deploy armies on cities. For safety, every city which has no army in it must has a neighboring city with two armies.
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Introduction

Origin

In ancient Roma, Emperor Constantine would deploy armies on cities. For safety, every city which has no army in it must has a neighboring city with two armies.

So

■ A Roman dominating function corresponds a way to protect all cities.

■ The weight of a Roman dominating function is the total number of armies.

■ The Roman domination number is the least number of armies that is enough to guard all cities.
Known results

1. $\gamma(G) \leq \gamma_R(G) \leq 2\gamma(G)$.
2. $\gamma_R(K_n) = 2$, $\gamma(K_n) = 1$ for $n \geq 2$.
3. $\gamma_R(C_n) = \gamma_R(P_n) = \lceil 2n/3 \rceil$, $\gamma_R(C_n) = \gamma(P_n) = \lceil n/3 \rceil$.
4. $\gamma_R(G) \leq 4n/5$, $\gamma(G) = n/2$ for every $G$ with minimum degree at least 1.
5. $\gamma_R(G) \leq 8n/11$, $\gamma(G) = 2n/5$ for every $G$ with minimum degree at least 2. (except 7 small graphs in domination case).
Chambers, Kinnersley, Prince and West [?] give the following conjecture.

**Conjecture:** For any 2-connected graph $G$ of $n$ vertices, $\gamma_R(G) \leq \lceil 2n/3 \rceil$.
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However, this conjecture isn’t true. We shall give infinitely many examples of 2-connected graphs with $\gamma_R(G) \geq 23n/34$ and prove the following theorem.

**Theorem**

For any 2-connected graph $G$ of $n$ vertices, $\gamma_R(G) \leq \max\{\lceil 2n/3 \rceil, 23n/34\}$.

Note that $23n/34 - 2n/3 = n/102$. 
Explosion Graphs

The *explosion graph* $G'$ of a multigraph $G$ is the graph obtained by replacing each edge $e = xy$ of the original graph by a 5-cycle $C_e = C_{xy}$ such that $x$ and $y$ are adjacent to two nonadjacent vertices in the 5-cycle respectively.

![Diagram of explosion graph](image)

**Figure**: Replace an edge $e = xy$ in $G$ by a 5-cycle $C_e = C_{xy}$.

We call $e_{x'}$, $e_{y'}$, $e_{xy}$ the *inner vertices* of $C_{xy}$ and of $G'$. 
Example: explosion graph of $K_4$
Examples of $\gamma_R(G) \geq 23n/34$

**Theorem**

There are infinitely many 2-connected graphs with Roman domination number at least $23n/34$, where $n$ is the number of vertices in the graph.
Examples of $\gamma_R(G) \geq 23n/34$

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**Proof:** Consider $k$ copies of graphs $G_1, G_2, \ldots, G_k$ each isomorphic to $K_4$, and their explosion graphs $G_1', G_2', \ldots, G_k'$. Let $G$ be a 2-connected graph obtained from the disjoint union of these explosion graphs $G_i'$'s by adding suitable edges between vertices of the original graphs $G_i$'s. Then, $G$ has $n = 34k$ vertices.
Examples of $\gamma_R(G) \geq 23n/34$ (continued)

Suppose to the contrary that $\gamma_R(G) < 23k$. Choose an optimal Roman dominating function $f$ of $G$. Since $\sum_{i=1}^{k} w(f, G'_i) = w(f) < 23k$, there is some $G'_i$ with $w(f, G'_i) < 23$. 
Examples of $\gamma_R(G) \geq 23n/34$ (continued)

Suppose to the contrary that $\gamma_R(G) < 23k$. Choose an optimal Roman dominating function $f$ of $G$. Since $\sum_{i=1}^{k} w(f, G'_i) = w(f) < 23k$, there is some $G'_i$ with $w(f, G'_i) < 23$.

For any edge $xy$ in $G_i$, no matter what are the values of $f(x)$ and $f(y)$, it is always the case that $w(f, C_{xy}) \geq 3$.

Furthermore, if $f(x) \leq 1$ and $f(y) \leq 1$, then $w(f, C_{xy}) \geq 4$. 
Examples of $\gamma_R(G) \geq 23n/34$ (continued)

Suppose to the contrary that $\gamma_R(G) < 23k$. Choose an optimal Roman dominating function $f$ of $G$. Since $\sum_{i=1}^{k} w(f, G'_i) = w(f) < 23k$, there is some $G'_i$ with $w(f, G'_i) < 23$.

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Furthermore, if $f(x) \leq 1$ and $f(y) \leq 1$, then $w(f, C_{xy}) \geq 4$.

Suppose $G_i$ has $r$ vertices $v$ with $f(v) \leq 1$, where $0 \leq r \leq 4$.

There are then $\binom{r}{2}$ edges $xy$ in $G_i$ with $w(f, C_{xy}) \geq 4$. Thus

$$23 > w(f, G'_i) \geq r \cdot 0 + (4 - r)2 + 6 \cdot 3 + \binom{r}{2},$$

which is impossible as $0 \leq r \leq 4$. □
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1. Introduction

2. Counter-examples

3. Roman domination on special graphs

4. Roman domination on 2-connected graphs

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Main idea

We often consider three Roman dominating functions $f_1$, $f_2$, and $f_3$ of $G$.

- We use $\vec{f}$ to denote the 3-tuple $(f_1, f_2, f_3)$, and $\vec{f}(v)$ for $(f_1(v), f_2(v), f_3(v))$.
- The weight of $\vec{f}$ is $w(\vec{f}) = \sum_{j=1}^{3} w(f_j)$. Note that $\gamma_R(G) \leq w(f_j) \leq w(\vec{f})/3$ for some $j$.
- A vertex $v$ is $\vec{f}$-strong if $f_j(v) = 2$ for some $j$.

Our goal is to construct $\vec{f}$ with $w(\vec{f}) \leq \max\{2n + 2, 69n/34\}$ for 2-connected graphs of $n$ vertices.
Example

$f$ is a 3-tuple Roman dominating function.

$w(f) = 4 + 3 + 3 = 10.$

Every vertex is $f$-strong.
For cycles

Lemma

If $n \geq 3$, then the $n$-cycle $C_n$ has a 3-tuple $\vec{f}$ of Roman dominating functions in which all vertices are $\vec{f}$-strong and $w(\vec{f}) \leq 2n$ when $n$ is a multiple of 3 and $w(\vec{f}) \leq 2n + 2$ otherwise.

Proof:

- For the case when $n = 3k$, the following $\vec{f}$ is as desired: $\vec{f}(v_{3i+1}) = (2, 0, 0)$, $\vec{f}(v_{3i+2}) = (0, 2, 0)$ and $\vec{f}(v_{3i+3}) = (0, 0, 2)$ for all $i$.
- When $n = 3k + 1$, modify $\vec{f}$ by changing $\vec{f}(v_1)$ to be $(2, 0, 1)$ and $\vec{f}(v_n)$ to be $(2, 1, 0)$.
- When $n = 3k + 2$, changing $\vec{f}(v_1)$ to be $(2, 0, 2)$.
For explosion graphs

**Theorem**

If \( G' \) is the explosion graph of a multigraph \( G \) without isolated vertices and \( G' \) has \( n' \) vertices, then \( G' \) has a 3-tuple \( \vec{f} \) of Roman dominating functions such that \( w(\vec{f}) \leq 69n'/34 \) and every non-inner vertex is \( \vec{f} \)-strong.

**Sketch of proof:** We shall assign \( \vec{f} \) on vertices of \( G \) first, and then assign the remaining vertices. For each \( \vec{f} \)-strong vertex \( x \), there exists \( j_x \) such that \( f_{j_x}(x) = 2 \). For each \( e = xy \in E(G) \), once we have assign \( \vec{f}(x) \) and \( \vec{f}(y) \), we can only guarantee that

- if \( j_x \neq j_y \), then \( w(\vec{f}, C_{xy}) \leq 3 + 3 + 4 = 2|C_{xy}|. \) (type-1)
- If \( j_x = j_y \), then \( w(\vec{f}, C_{xy}) \leq 3 + 4 + 4 = 2|C_{xy}| + 1. \) (type-2)
We shall assign $\vec{f}$ greedly.

1. Order vertices of $G$ into $v_1, v_2, \ldots, v_n$ according to their degree and number of neighbors.

2. Assign $\vec{f}(v_i)$ according to $\vec{f}(v_k)$, where $k < i$ and $v_k$ is adjacent to $v_i$.

3. For every edge $e = xy \in E(G)$, assign $\vec{f}$ on $C_{xy}$ according to $\vec{f}(x)$ and $\vec{f}(y)$.

Note that $69/34 = 2 + 1/34$. We only need to promise that type-2 edges only occur for each 4 vertices and 6 edges in $G$. (i.e. 34 vertices in $G'$.) □
Adding a path

Lemma

Suppose $G$ has a 3-tuple $\vec{f}$ of Roman dominating functions for which $u$ and $v$ are $\vec{f}$-strong. If $G'$ is obtained from $G$ by adding a disjoint path $P = v_1 v_2 \ldots v_t$ with $t \geq 1$ and two edges $uv_1$ and $v_t v$, then $\vec{f}$ can be extended to $G'$ such that $w(\vec{f}, P) = 2t$ and $v_i$ is $\vec{f}$-strong for $1 < i < t$.

Proof: Assume that $f_j(u) = 2$ and $f_k(v) = 2$. We shall define $\vec{f}$ by: $\vec{f}(v_{3i+1}) = (2, 0, 0)$, $\vec{f}(v_{3i+2}) = (0, 2, 0)$ and $\vec{f}(v_{3i+3}) = (0, 0, 2)$ for all $i$ with some modifications according to the value $(t \mod 3)$ and whether $j = k$ or not in following six cases.
Adding a path (continued)

**Case 1.** \( t \equiv 0 \pmod{3} \) and \( j = k \), say \( j = k = 1 \). In this case, change \( \vec{f}(v_1) \) from \((2, 0, 0)\) to \((0, 0, 1)\) and \( \vec{f}(v_2) \) from \((0, 2, 0)\) to \((1, 2, 0)\) as follows.

\[
\begin{array}{cccccccc}
 u & v_1 & v_2 & v_3 & \cdots & v_{t-2} & v_{t-1} & v_t & v \\
 f_1 & 2 & 0 & 1 & 0 & \cdots & 2 & 0 & 0 & 2 \\
 f_2 & * & 0 & 2 & 0 & \cdots & 0 & 2 & 0 & * \\
 f_3 & * & 1 & 0 & 2 & \cdots & 0 & 0 & 2 & * \\
\end{array}
\]

**Case 2.** \( t \equiv 0 \pmod{3} \) and \( j \neq k \), say \( j = 3 \) and \( k = 1 \). In this case, no modification is needed.

\[
\begin{array}{cccccccc}
 u & v_1 & v_2 & v_3 & \cdots & v_{t-2} & v_{t-1} & v_t & v \\
 f_1 & * & 2 & 0 & 0 & \cdots & 2 & 0 & 0 & 2 \\
 f_2 & * & 0 & 2 & 0 & \cdots & 0 & 2 & 0 & * \\
 f_3 & 2 & 0 & 0 & 2 & \cdots & 0 & 0 & 2 & * \\
\end{array}
\]
**Case 3.** \( t \equiv 1 \pmod{3} \) and \( j = k \), say \( j = k = 1 \). In this case, if \( t = 1 \), then change \( \vec{f}(v_1) \) from \((2, 0, 0)\) to \((0, 1, 1)\) as follows.

\[
\begin{array}{ccc}
  u & v_1 & v \\
  f_1 & 2 & 0 \\
  f_2 & * & 1 \\
  f_3 & * & 1 \\
\end{array}
\]

As for \( t \neq 1 \), change \( \vec{f}(v_1) \) from \((2, 0, 0)\) to \((0, 0, 1)\), \( \vec{f}(v_2) \) from \((0, 2, 0)\) to \((1, 2, 0)\), \( \vec{f}(v_{t-1}) \) from \((0, 0, 2)\) to \((1, 0, 2)\) and \( \vec{f}(v_t) \) from \((2, 0, 0)\) to \((0, 1, 0)\) as follows.

\[
\begin{array}{cccccccc}
  u & v_1 & v_2 & v_3 & \cdots & v_{t-2} & v_{t-1} & v_t & v \\
  f_1 & 2 & 0 & 1 & 0 & \cdots & 0 & 1 & 0 & 2 \\
  f_2 & * & 0 & 2 & 0 & \cdots & 2 & 0 & 1 & * \\
  f_3 & * & 1 & 0 & 2 & \cdots & 0 & 2 & 0 & * \\
\end{array}
\]
Adding a path (continued)

**Case 4.** $t \equiv 1 \pmod{3}$ and $j \neq k$, say $j = 3$ and $k = 2$. In this case, no modification is needed.

$$
\begin{array}{cccccccc}
  u & v_1 & v_2 & v_3 & \cdots & v_{t-2} & v_{t-1} & v_t & v \\
  f_1 & * & 2 & 0 & 0 & \cdots & 0 & 0 & 2 & * \\
  f_2 & * & 0 & 2 & 0 & \cdots & 2 & 0 & 0 & 2 \\
  f_3 & 2 & 0 & 0 & 2 & \cdots & 0 & 2 & 0 & * \\
\end{array}
$$

**Case 5.** $t \equiv 2 \pmod{3}$ and $j = k$, say $j = k = 3$. In this case, no modification is needed.

$$
\begin{array}{cccccccc}
  u & v_1 & v_2 & v_3 & \cdots & v_{t-2} & v_{t-1} & v_t & v \\
  f_1 & * & 2 & 0 & 0 & \cdots & 0 & 2 & 0 & * \\
  f_2 & * & 0 & 2 & 0 & \cdots & 0 & 0 & 2 & * \\
  f_3 & 2 & 0 & 0 & 2 & \cdots & 2 & 0 & 0 & 2 \\
\end{array}
$$
Case 6. $t \equiv 2 \pmod{3}$ and $j \neq k$, say $j = 1$ and $k = 3$. In this case, change $\vec{f}(v_1)$ from $(2, 0, 0)$ to $(0, 0, 1)$ and $\vec{f}(v_2)$ from $(0, 2, 0)$ to $(1, 2, 0)$ as follows.

$$
\begin{array}{cccccccc}
& u & v_1 & v_2 & v_3 & \cdots & v_{t-2} & v_{t-1} & v_t & V \\
 f_1 & 2 & 0 & 1 & 0 & \cdots & 0 & 2 & 0 & \star \\
f_2 & \star & 0 & 2 & 0 & \cdots & 0 & 0 & 2 & \star \\
f_3 & \star & 1 & 0 & 2 & \cdots & 2 & 0 & 0 & 2 \\
\end{array}
$$

Note that $v_1$ and $v_t$ are also $\vec{f}$-strong for Case 2, 4 and 5.
Adding a tailed cycle

A tailed $t$-cycle $H$ is a cycle $v_1v_2\ldots v_tv_1$ together with a path $u_1u_2\ldots u_s$ call the tail and an edge $u_sv_1$. We call $u_1$ the starting vertex and $v_t$ the inner vertex of $H$.

**Lemma**

Suppose $G$ has a 3-tuple $\vec{f}$ of Roman dominating functions for which $u$ is $\vec{f}$-strong. If $G'$ is obtained from $G$ by adding a tailed $t$-cycle $H$ with $t \equiv 1 \pmod{3}$ and an edge $uu_1$, then $\vec{f}$ can be extended to $G'$ such that $w(\vec{f},H) = 2|V(H)|$ and all vertices of $H$ except the inner vertex are $\vec{f}$-strong.
Proof: Without loss of generality, we may assume that $f_3(u) = 2$. First, we define $\vec{f}$ for the vertices on the tail of $H$ as $\vec{f}(u_{3i+1}) = (2, 0, 0)$, $\vec{f}(u_{3i+2}) = (0, 2, 0)$ and $\vec{f}(u_{3i+3}) = (0, 0, 2)$ for all $i$.

Without loss of generality, we may assume that $f_1(u_s) = 2$.

Next, define $\vec{f}$ for the other vertices of $H$ as $\vec{f}(v_{3i+1}) = (0, 2, 0)$, $\vec{f}(v_{3i+2}) = (0, 0, 2)$ and $\vec{f}(v_{3i+3}) = (2, 0, 0)$ for all $i$ with the modifications of changing $\vec{f}(v_{t-1})$ from $(2, 0, 0)$ to $(2, 1, 0)$ and changing $\vec{f}(v_t)$ from $(0, 2, 0)$ to $(0, 0, 1)$. The extension is then as desired. \qed
Adding a tailed $\theta$-graph

A *tailed* $\theta$-graph $H$ consists of three internally disjoint paths $P_i : av_{i,1}v_{i,2} \ldots v_{i,t_i}b$, where $t_i$ is a multiple of 3 for $1 \leq i \leq 3$, together with a path $u_1u_2\ldots u_s$ call the *tail* and an edge $u_s a$. We call $u_1$ the *starting* vertex and $v_{1,1}$, $v_{2,1}$ and $v_{3,1}$ the *inner* vertices of $H$.

**Lemma**

*Suppose $G$ has a 3-tuple $\vec{f}$ of Roman dominating functions for which $u$ is $\vec{f}$-strong. If $G'$ is obtained from $G$ by adding a disjoint tailed $\theta$-graph $H$ and an edge $uu_1$, then $\vec{f}$ can be extended to $G'$ such that $w(\vec{f}, H) = 2|V(H)|$ and all vertices of $H$ except the inner vertices are $\vec{f}$-strong.*
Adding a tailed $\theta$-graph (continued)

**Proof:** Without loss of generality, we may assume that $f_3(u) = 2$. First, we define $\vec{f}$ for the vertices on the tail of $H$ as $\vec{f}(u_{3i+1}) = (2, 0, 0)$, $\vec{f}(u_{3i+2}) = (0, 2, 0)$ and $\vec{f}(u_{3i+3}) = (0, 0, 2)$ for all $i$. Without loss of generality, we may assume that $f_2(u_s) = 2$. Next, define $\vec{f}$ for the vertices of $P_k$ as $\vec{f}(v_{k,3i+1}) = (2, 0, 0)$, $\vec{f}(v_{k,3i+2}) = (0, 2, 0)$ and $\vec{f}(v_{k,3i+3}) = (0, 0, 2)$ with the modification of changing $\vec{f}(v_{k,1}) = (0, 0, 0)$, $\vec{f}(v_{k,2}) = (1, 2, 0)$ for all $k = 1, 2, 3$ and $i$. Finally, define $\vec{f}(a) = (2, 0, 2)$ and $\vec{f}(b) = (2, 1, 0)$. Then, $\vec{f}$ is as desired. $\square$
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**Main theorem**

<table>
<thead>
<tr>
<th>Theorem</th>
</tr>
</thead>
<tbody>
<tr>
<td>If $G$ is a 2-connected graph of $n$ vertices, then $\gamma_R(G) \leq \max{\lceil2n/3\rceil, 23n/34}$.</td>
</tr>
</tbody>
</table>

We shall prove the theorem by contradiction. Assume that $G$ is a *minimum counter-example* to the theorem. In other words, $G$ is a 2-connected graph of order $n$ with $\gamma_R(G) > \max\{\lceil2n/3\rceil, 23n/34\}$, and any 2-connected graph $G'$ of order $n'$ with $n' + |E(G')| < n + |E(G)|$ satisfies $\gamma_R(G') \leq \max\{\lceil2n'/3\rceil, 23n'/34\}$.
Proof of the main theorem

Suppose the theorem is not true. Find a minimum counter example $G$. We shall start with a subgraph $L$ of $\ell$ vertices and construct a 3-tuple of Roman dominating functions $\vec{f}$ of $L$ for which all boundary vertices of $L$ are $\vec{f}$-strong and $w(\vec{f}, L) \leq \max\{2\ell + 2, 69\ell/34\}$. If $\ell = n$ then we reach a contraction. Otherwise we add to $L$ a subgraph $H$ of $G - V(L)$ with $h$ vertices and some edges between $H$ and $L$ to get a new $L'$ of $\ell' = \ell + h$ vertices; and extend $\vec{f}$ to be a 3-tuple of Roman dominating function of $L'$ such that all boundary vertices of $L'$ are $\vec{f}$-strong and $w(\vec{f}, H) \leq 2h$ or equivalently $w(\vec{f}, L') \leq \max\{2\ell' + 2, 69\ell'/34\}$. We then replace $L$ by $L'$ and continue the process until $\ell = n$, which leads to a contradiction.
Suppose $L$ is a subgraph of $G$, a vertex in $L$ is called a boundary (respectively, interior) vertex with respect to $G$ if it is adjacent to some (respectively, no) vertex in $V(G) - V(L)$.

**Lemma**

Suppose $G$ is a minimum counter-example to Theorem ??, and $L$ is a connected subgraph of $G$ which has a 3-tuple $\vec{f}$ of Roman dominating functions such that boundary vertices of $L$ are $\vec{f}$-strong. If $V(L) \neq V(G)$ and $G - V(L)$ has no 3p-cycle and explosion graph whose inner vertices are of degree 2 in $G$ as subgraphs, then one of the following statements holds.
Key lemma (continued)

1. $G - V(L)$ has a path $H$ whose endpoints are adjacent to vertices in $L$ and are also interior vertices of $L \cup H$.
2. $G - V(L)$ has a tailed $(3p + 1)$-cycle $H$ whose starting vertex is adjacent to a vertex in $L$ and whose inner vertex is of degree 2 in $G$.
3. $G - V(L)$ has a tailed $\theta$-graph $H$ whose starting vertex is adjacent to a vertex in $L$ and whose inner vertices are of degree 2 in $G$.
4. $G - V(L)$ has a $3p$-path or a $(3p + 1)$-path $H$ whose endpoints adjacent to some $u, v$ in $L$ and $f_i(u) = f_j(v) = 2$ for some $i \neq j$.
5. $G - V(L)$ has a $(3p + 2)$-path $H$ whose endpoints adjacent to some $u, v$ in $L$ and $f_i(u) = f_i(v) = 2$ for some $i$. 
Proof of the main theorem (continued)

Step 1. Find a subgraph $L$ of $G$ with as many number of vertices as possible such that $L$ is the explosion graph of some graph without isolated vertices and the inner vertices of $L$ are not boundary vertices. By Theorem ??, a desired $\vec{f}$ exists. If there is no such graph, just view $L$ as an empty graph.

Step 2. Find a cycle $H$ of length a multiple of 3 in $G - V(L)$. By Lemma ??, $\vec{f}$ can be extended to $L'$ as desired. Repeat this step until no more such cycle exists.
Proof of the main theorem (continued)

Step 3. If $L$ is not connected, then choose two components and a path $H$ in $G - V(L)$ with one endpoint adjacent to a vertex in a component and the other endpoint adjacent to a vertex in the other component. By Cases 2, 4, 5 of Lemma ??, $\vec{f}$ can be extended to $L'$ as desired. Notice that in order to use Cases 2, 4, 5 we may have to inter-change the role of $f_j$’s in a component so that we can really apply these three cases. Repeat this step until $L$ is connected.

Step 4. If Steps 1 to 3 result an empty graph, then find a cycle as $L$, which is as desired by Lemma ??.
Proof of the main theorem (continued)

After performing Steps 1 to 4, we have a desired induced subgraph $L$ which is also connected. By Lemma ??, we can always find a subgraph $H$ from $G - V(L)$ satisfying one of the five cases. We then can use Lemmas ??, ?? and ?? to extend $\vec{f}$ to $L'$ as desired. □
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Reference (continued)


Reference (continued)


N. Prince, *Thresholds for Roman domination*, manuscript.


THANK YOU