Roman Domination on Graphs
Gerard Jennhwa Chang
Department of Mathematics, National Taiwan University

Recent articles by ReVelle [20, 21] in the Johns Hopkins Magazines suggested a new variation of domination called Roman domination, see also [22] for an integer programming formulation of the problem. Since then, there have been several articles on Roman domination and its variations [2, 3, 4, 5, 6, 11, 12, 14, 15, 16, 18, 24, 23, 25]. Emperor Constantine had the requirement that an army or legion could be sent from its home to defend a neighboring location only if there was a second army which would stay and protect the home. Thus, there are two types of armies, stationary and travelling. Each vertex (city) has no army must have a neighboring vertex with a travelling army. Stationary armies then dominate their own vertices, and a vertex with two armies is dominated by its stationary army, and its open neighborhood is dominated by the travelling army.

The concept of Roman domination can be formulated in terms of graphs. We consider only simple graph $G$ with vertex set $V(G)$ and edge set $E(G)$. The (open) neighborhood of a vertex $v \in V(G)$ is defined as $N(v) = \{u : uv \in E(G)\}$. A Roman dominating function of a graph $G$ is a function $f : V(G) \rightarrow \{0, 1, 2\}$ such that whenever $f(v) = 0$, there exists a vertex $u \in N(v)$ for which $f(u) = 2$. The weight of $f$ is $\sum_{v \in V(G)} f(v)$. The Roman domination number $\gamma_R(G)$ of $G$ is the minimum weight of Roman dominating functions of $G$.

In this talk, we first survey known results, including those in Table 1 where we denote the number of vertices, the minimum degree and the domination number of $G$ as $n$, $\delta(G)$ and $\gamma(G)$, respectively.

| $\gamma(G)$ | $\gamma_R(G)$ | $2\gamma(G)$ | $\gamma(G) = \gamma_R(G)$ | $\gamma_R(G)$ implies that vertices of $G$ forms an independent set | $\gamma_R(G) \leq 4n/5$ for $G$ of $n$ vertices and $\delta(G) \geq 1$ | $\gamma_R(G) \leq 8n/11$ for $G$ of $n$ vertices and $\delta(G) \geq 2$ | $\gamma_R(G) \leq 2n / \delta(G) + 1 + 1 / 2$ | $\gamma_R(P_n) = \gamma_R(C_n) = \lceil 2n/3 \rceil$ | Linear-time algorithm for Roman domination on interval graphs | Linear-time algorithm for Roman domination on co-graphs | Polynomial-time algorithm for Roman domination on AT-free graphs |

I will also present some recent results by Chun-Hung Liu and myself. These include a unified approach to the Roman domination problem and its three variations on interval graphs, a linear-time algorithm to the $(a, b)$-Roman domination problem on strongly chordal graphs, as well as a result about a conjecture by Chambers, Kinnersley, Prince and West [6].
References


