A note on Path Kernels and Partitions

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Abstract

The detour order of a graph $G$, denoted by $\tau(G)$, is the order of a longest path in $G$. A subset $S$ of $V(G)$ is called a $P_n$-kernel of $G$ if $\tau(G[S]) \leq n - 1$ and every vertex $v \in V(G) - S$ is adjacent to an end-vertex of a path of order $n - 1$ in $G[S]$. A partition of the vertex set of $G$ into two sets, $A$ and $B$, such that $\tau(G[A]) \leq a$ and $\tau(G[B]) \leq b$ is called an $(a, b)$-partition of $G$. In this paper we show that any graph with girth $g$ has a $P_{n+1}$-kernel for every $n < \frac{3g}{2} - 1$. Furthermore, if $\tau(G) = a + b$, $1 \leq a \leq b$, and $G$ has girth greater than $\frac{3}{\frac{3}{2}(a + 1)}$, then $G$ has an $(a, b)$-partition.

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