Coloring Games on the Square of Graphs

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Abstract

The game chromatic number and game coloring number of the square of graphs were first studied by Esperet and Zhu in [1]. In [1], Esperet and Zhu showed that if \( G \) is a forest of maximum degree \( \Delta \geq 9 \), then \( \chi_g(G^2) \leq \text{col}_g(G^2) \leq \Delta + 3 \), and there are forests \( G \) with \( \text{col}_g(G^2) = \Delta + 3 \). It is also shown in [1] that for an outerplanar graph \( G \) of maximum degree \( \Delta \), \( \chi_g(G^2) \leq \text{col}_g(G^2) \leq 2\Delta + 16 \), and for a planar graph \( G \) of maximum degree \( \Delta \), \( \chi_g(G^2) \leq \text{col}_g(G^2) \leq 23\Delta + 41 \).

In this paper, we show that if \( G \) is a partial \( k \)-tree and \( a < k \), then \( (a, 1)\text{-gcol}(G^2) \leq k\Delta + (1 + \frac{1}{a})(\frac{k^2 + 3k + 2}{2}) + 2 \); Especially for partial \( k \)-trees, \( \text{gcol}(G^2) \leq k\Delta + k^2 + 3k + 4 \). For planar graphs \( G \), there exists a constant \( C \), such that \( \text{gcol}(G^2) \leq 5\Delta + C \).

For a graph \( G \), the maximum average degree of \( G \) is defined as \( \text{Mad}(G) = \max\{\frac{2|E(H)|}{|V(H)|} : H \text{ is a subgraph of } G\} \). The very asymmetric coloring games on the square of graphs with \( a \geq k \), where \( \text{Mad}(G) \leq 2k \), were also studied in this paper.

References


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