On the Equitable Coloring of Kneser Graphs

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Abstract

An $m$-coloring of a graph $G$ is a mapping $f : V(G) \to \{1, 2, \ldots, m\}$ such that $f(x) \neq f(y)$ for any two adjacent vertices $x$ and $y$ in $G$. The chromatic number $\chi(G)$ of $G$ is the minimum number $m$ such that $G$ is $m$-colorable. An equitable $m$-coloring of a graph $G$ is an $m$-coloring $f$ such that any two color classes differ in size by at most one. The equitable chromatic number $\chi_=(G)$ of $G$ is the minimum number $m$ such that $G$ is equitably $m$-colorable. The equitable chromatic threshold $\chi^\star_=(G)$ of $G$ is the minimum number $m$ such that $G$ is equitably $r$-colorable for all $r \geq m$. It is clear that $\chi(G) \leq \chi_=(G) \leq \chi^\star_=(G)$.

For $n \geq 2k + 1$, the Kneser graph $KG(n, k)$ has the vertex set consisting of all $k$-subsets of an $n$-set. Two distinct vertices are adjacent in $KG(n, k)$ if they have empty intersection as subsets. The Kneser graph $KG(2k + 1, k)$ is called the Odd graph, denoted by $O_k$. In this talk, we study the equitable colorings of Kneser graphs $KG(n, k)$. Mainly, we obtain that $\chi_=(KG(n, k)) \leq \chi^\star_=(KG(n, k)) \leq n - k + 1$ and $\chi(O_k) = \chi_=(O_k) = \chi^\star_=(O_k) = 3$. We also show that $\chi_=(KG(n, k)) = \chi^\star_=(KG(n, k))$ for $k = 2$ or $3$ and obtain their exact values.

Keywords: equitable coloring, equitable chromatic number, equitable chromatic threshold, Kneser graph, odd graph, intersection family.

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